SUPERCONDUCTING ALTERNATORS

FOR MARINE PROPULSION

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Thesis Supervisor: H. H. Woodson
Title: Professor of Electrical Engineering

June, 1967

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by

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ABSTRACT

This paper develops the field and energy equations and the inductance parameters for an alternator with a rotating, superconducting field, a stationary, normal armature and an iron shield enclosing the outer radius of the alternator. The purpose of this paper was to indicate the feasibility of a superconducting alternator being used for marine propulsion. A 27,200 kw generator is designed with a total weight of 4.65 tons, a diameter of .754 meters, and a length of 1.2 meters. On the basis of the weight and volume saved, superconducting alternators are feasible for marine propulsion.

It is recommended that more work be done to develop a procedure to optimize the design of an alternator. The rest of the propulsion system needs to be developed and a model built to test out the design.

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LIST OF SYMBOLS

a,f	Subscripts for armature and field
a,b,c	Designation of phases
d	Diameter
$\bar{k}$	Average heat transfer coefficient
kw	Kilowatts
l	Electrical length of alternator
l_a	Physical length of alternator
n	Order of harmonic
n	Rotational speed, rpm
p	Number of pole pairs
r	Radius to a differential cylinder of current
t	Thickness
x	Ratio of inside to outside radius of armature, $\frac{R_1}{R_o}$
y	Ratio of inside to outside radius of field, $\frac{R_1}{R_2}$
w	Ratio of inside radius of shield to outside radius of armature, $\frac{R_s}{R_o}$
z	Ratio of inside radius of shield to outside radius of field, $\frac{R_s}{R_2}$
B	Magnetic field intensity
H, H_ρ, H_ϕ	Magnetizing force
H	Horsepower
I_a, I_f	Armature and field currents
J_a, J_f	Armature and field current density
K_R	Surface current density on cylinder of radius r
L_a, L_f	Inductance of armature and field

L_{af}	Mutual inductance between armature and field
M	Magnitude of first order mutual inductance
N_a, N_f	Number of turns in armature and field
R_1	Inside radius of field
R_2	Outside radius of field
R_i	Inside radius of armature
R_o	Outside radius of armature
R_s	Inside radius of shield
R_{so}	Outside radius of shield
T	Temperature in ^{o}K
W_a, W_f	Energy in armature and field as magnetic field
λ	Flux linkage
ψ	Dummy variable for integration
φ	Cylindrical angle to a specific point
ρ	Radius to a specific point
μ_o	Permeability of free space
τ	Stress
ω	Electrical Angular speed
ω_m	Mechanical angular speed

I. INTRODUCTION

Electric propulsion plants find little use in ships today. By electric propulsion is meant that the prime mover drives an electrical generator and a motor drives the shaft of the propulsion device. Two of the prime reasons are that the specific weight and specific volume of electrical plants are significantly greater than those of a propulsion system using a shaft and reduction gears. Another reason is that the initial cost of an electrical plant is greater than for other plants. Electrical plants have a number of advantages which, if the size and weight could be made to compare with existing plants, would put them in a competitive position with the existing plants. One of the primary advantages is the better response and control that is achievable with an electric plant. An electric plant makes bridge control possible. Another advantage is the freedom of arrangement that an electrical plant allows. The absence of shafts from amidships to the stern permits fuller use of the spaces located there. The machinery arrangement is not dictated by the need for a reduction gear set to be attached to the end of a shaft. With a small alternator the arrangements would be even more versatile. A gas turbine and generator could be placed in any convenient place on the ship. Electrical plants operate at a high efficiency. By running the turbine-generator at a constant speed, the best efficiency of the turbine can be achieved. There are fewer and less complex moving parts in an electrical system. This means less of a maintance problem. Also with no direct connection from the machinery space to the propeller, there is no direct path for machinery noise to be transmitted to the sea.

Conventional electric plants for a given output horsepower are about four times the weight of a set of reduction gears, shafting and associated equipment for the same output. With the use of superconductors in electric machines, the size and weight can be greatly reduced. Superconductors are materials which lose all resistance near absolute zero. The references [3, 4, 5] describe individual characteristics.

In "Superconductivity: Status and Implication for Marine Applications", by E. H. Sibley, E. G. Frankel, and J. M. Reynolds [2], the size of superconducting marine power plants is estimated. The paper makes no mention of shielding the superconducting plant to protect operating personnel and to keep the fields from interfering with the operation of other machinery in the area. It is not feasible to isolate the superconducting machinery from the rest of the ship. Space is too critical to do that.

Dynatech Corporation, Cambridge, Massachusetts; Avco-Everett, Everett, Massachusetts, and other corporations have designed small superconducting alternators. None of these have shielding and none are large enough to be of use for marine propulsion.

It is the purpose of this paper to develop the equations for the model of a shielded superconducting alternator and to evaluate the model with values to produce 35,000 shaft horsepower output. This is approximately the output of a destroyer plant. Other work which will have to be done to complete the electrical plant is to design a control system, design the coolant plant, design the power leads and do the structural design of the turbine and rotating electrical equipment.

Assumptions and Description

Figure 1 shows the arrangement of the alternator. The field is on the inside and is rotating. It is the portion of the machine which will be superconducting. The armature and shield are fixed and located around the field. This is the simplest arrangement since the power must be taken off the armature. It would be impractical to take that much power off on slip rings. Little power is needed to operate the field as it is superconducting. The armature current density assumed is 100 amps/cm^2. This allows a packing factor of .25. A density of 10,000 amps/cm^2 is assumed for the superconducting field winding. This allows at least the same packing factor as it is conservative as to the current carrying capacity of the superconductor.

The system is composed of a prime mover driving the superconducting generator. The power from the generator goes to a control device, possibly a cycloconverter*, which will change the voltage and frequency to allow variable output speed and torque of the superconducting motor which it drives. The response of the system only depends on the dynamic response of the motor, shaft and propeller. This should permit much better response times than reduction gear and shaft drives.

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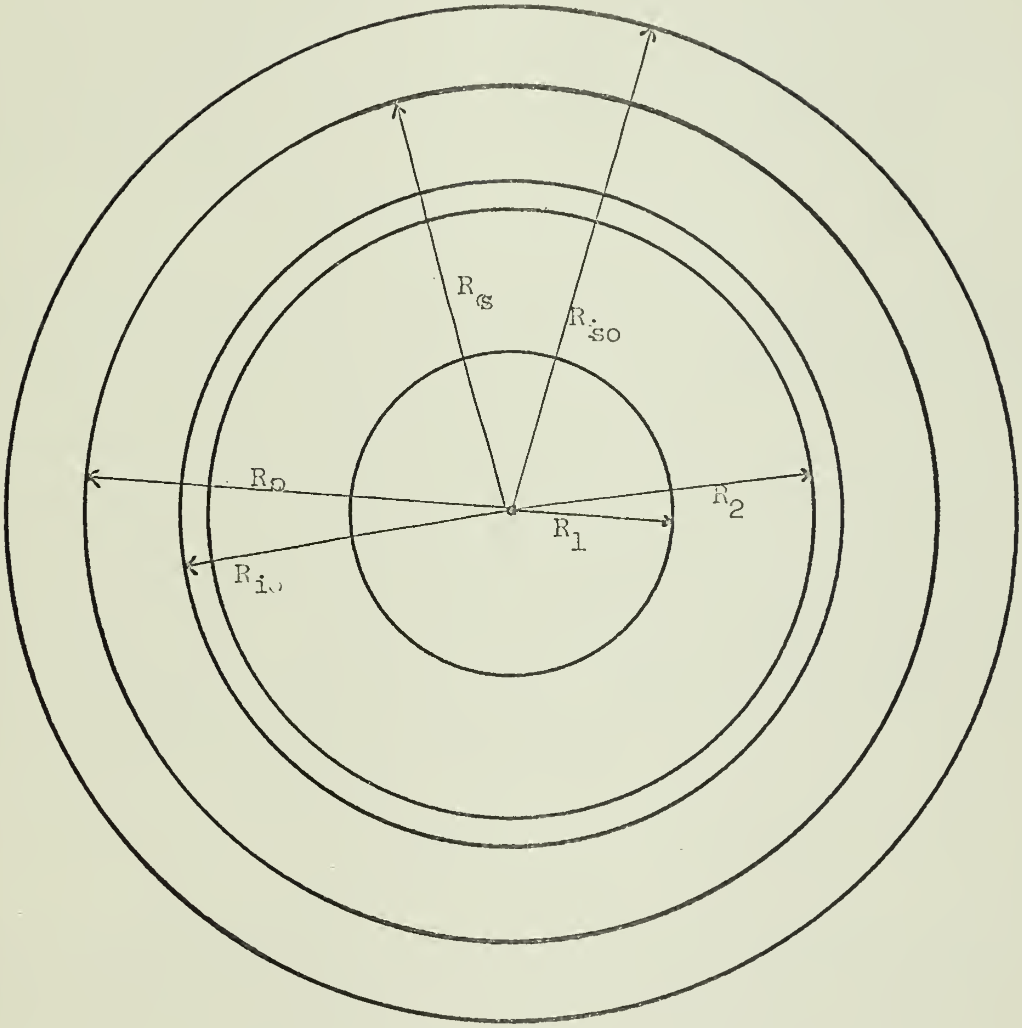
*A cycloconverter is an electronic device which accepts power at the input voltage and frequency and converts it to a voltage and frequency determined by its control signal.

FIGURE 1

CROSS SECTION OF ALTERNATOR



R_1	inside radius of field
R_2	outside radius of field
R_i	inside radius of armature
R_o	outside radius of armature
R_s	inside radius of shield
R_{so}	outside radius of shield

II. DEVELOPMENT OF MODEL

The model will be developed in three parts. The fields generated by the field winding will be considered first. Then the fields generated by the armature will be developed. The last part will be to add the effects of the shield on the fields. All fields are the solution to the equation

$$\nabla^2 U = 0 \tag{II-1}$$

subject to the appropriate boundary conditions. The detailed development of the model will be found in Appendix A. This chapter presents a discussion of the procedure and a presentation of the results.

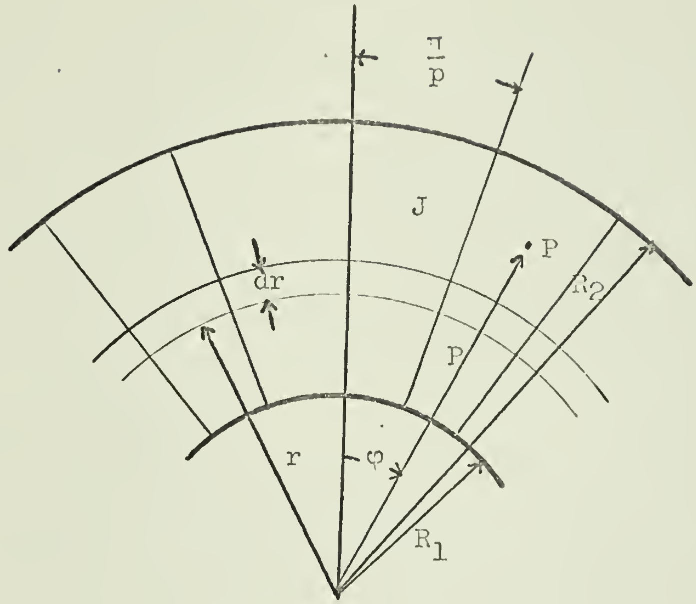
The fields are essentially independent of the axial dimension. This is not true in the end turns but their contribution to the machine is small and will be neglected. Therefore the solutions to Laplace's Equation are in terms of radius and cylindrical angle. They can be expressed as the product of $R(\rho)$ and $\Phi(\varphi)$, where R and Φ are

$$R = A\rho^q + B\rho^{-q}$$

$$\Phi = C\sin q\varphi + D\cos q\varphi \tag{II-2}$$

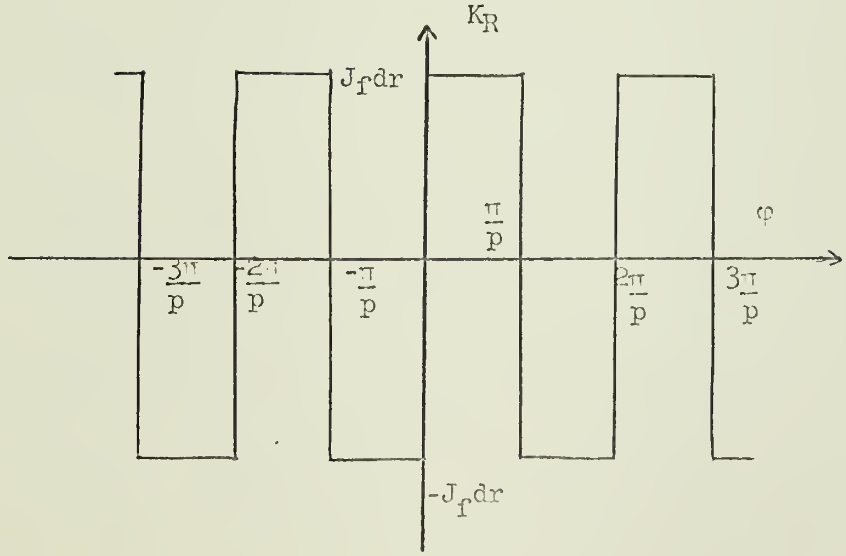
with the constants properly evaluated to satisfy the boundary conditions in each situation. The boundary conditions applicable to this problem are that the solution must at all points be bounded, must be zero in the shield, and must satisfy the current distribution in the field windings and the armature windings. Since the magnitude of the fields is to be constrained to that which will produce no saturation in the materials used, the solutions can be solved independently and superimposed. Figure 2 shows the physical distribution of the current density in the field windings.

FIGURE 2



p pole-pair field winding

FIGURE 3



Distribution of surface current density in shell of thickness dr for field winding [1].

The point $P(\rho, \varphi)$ is the point for which the field is being computed due to the current density shell dr thick at radius r. Figure 3 shows the angular distribution of the current density in the shell dr thick. The value of q is readily found as the angular dependence must be the same in the field as it is in the distribution of the current. Fourier analysis of the current density distribution yeilds

$$K(r,\varphi) = \sum_{n\ \mathrm{odd}} \frac{4J_f dr \sin np\varphi}{n\pi} \qquad \text{(II-3)}$$

therefore q is np. The general solution is

$$U(r,\varphi) = R(r)\Phi(\varphi) = (A\rho^{np} + B\rho^{-np})(C\sin np\varphi + D\cos np\varphi) \qquad \text{(II-4)}$$

The field components are

$$H\rho = \frac{\partial U}{\partial \rho} \qquad H\varphi = \frac{1}{\rho}\frac{\partial U}{\partial \varphi} \qquad \text{(II-5)}$$

and the boundary conditions are

$H(\rho=o)$ is finite

$H(\rho\to\infty) = 0$

$$\frac{\partial H\rho}{\partial \rho} = 0 \quad (\rho=r)$$

$$\frac{\partial H\varphi}{\partial \rho} = K(r,\varphi) \quad (\rho=r) \qquad \text{(II-6)}$$

After the field solution is found in terms of r, φ, and ρ, an integration from R_1 to R_2 is performed to sum all the contributing rings of surface current. There are field components from the shielding which come from the general solution, equation II-4, but with the boundary conditions that $H(\rho=o)$ is finite and H_φ of the shield solution at $\rho=R_s$ is the negative of its field counterpart so that the total $H\varphi$ is zero at $\rho=R_s$. The shield field is added to the field to obtain the total field inside the shield.

The results are shown in Table I. The armature field can be solved by the same procedure. The current distribution is different as shown in Figures 4 and 5. Fourier analysis of the current distribution yields

$$K(r,\varphi) = \sum_{n\ \mathrm{odd}} \frac{4J_a dr}{n\pi} \cos\frac{n\pi}{3} \sin np\varphi \tag{II-7}$$

The general solution has the same form and the field components are obtained the same way. The solution is evaluated the same way except that the result is integrated from R_i to R_o to remove the r dependence. Again the shield components to the armature field are added in to obtain the total field which is also zero at $r = R_s$. These results are shown in Table II. Notice that each entry in the tables is of the form A(1+B). A is the solution without the shield and B is the factor which, when multiplied by A, is the contribution of the shield.

The energy in the two fields is obtained from the equation

$$W = l \int_0^{2\pi} \int_0^{R_{si}} 1/2\, \mu_o \left(H_\rho^2 + H_\varphi^2\right) \rho\, d\rho\, d\varphi \tag{II-8}$$

From the field energies, the self inductances can be found.

$$L = \frac{W}{I^2/2} \tag{II-9}$$

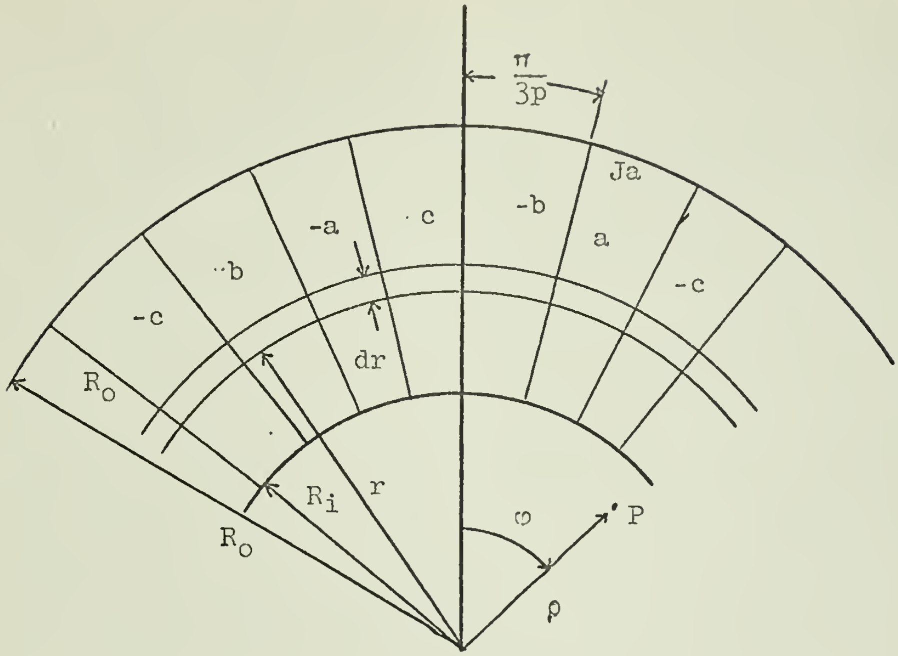
The mutual inductance is obtained from

$$L_{af} = \frac{\lambda}{I_f} \tag{II-10}$$

where the second derivative of λ is the product of the number of turns in an elemental coil of the armature and the field of the stator linking those turns.

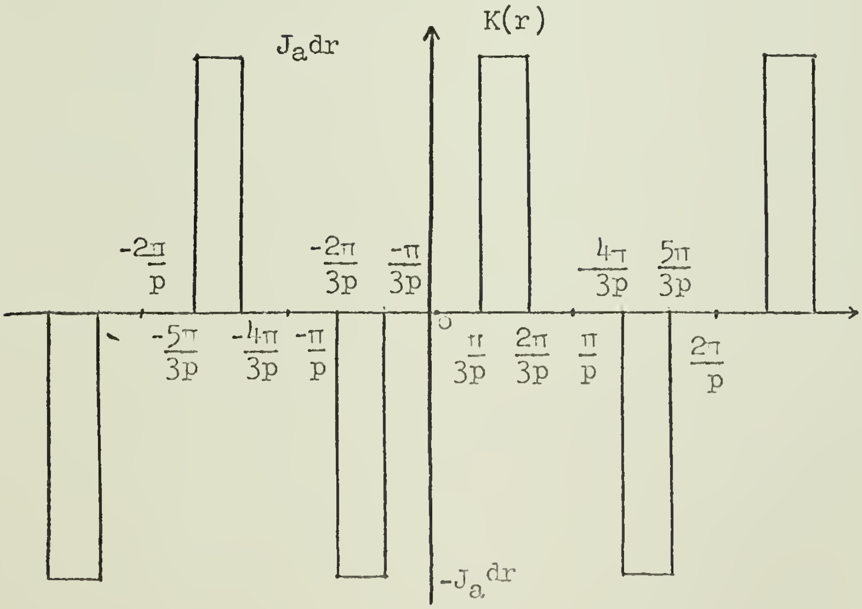
$$d^2\lambda = \frac{6N_a \rho\, d\rho\, d\varphi}{\pi(R_o^2 - R_i^2)} \mu_o \int_{\varphi - \frac{\pi}{p}}^{\varphi} H_\rho(\rho,\varphi) \ldots \tag{II-11}$$

FIGURE 4



P pole-pair, 3 phase armature winding

FIGURE 5



Distribution of phase a surface current density in shell of thickness dr for armature winding [1].

This equation is then integrated twice to obtain λ. The maximum value of the primary component of L_{af} is called M. All the inductances are listed in Table III.

TABLE I

Field Intensity of Field Winding with Shielding $\quad np \neq 2$

$\rho < R_1$

$$H_\rho = \sum_{n\ \text{odd}}' \frac{2Jf}{\pi(2-np)n} \left(R_2^{-np+2} - R_1^{-np+2}\right) \rho^{np-1} \cos np\varphi$$

$$\left[1 + \frac{2-np}{2+np} \frac{1}{R_s^{2np}} \frac{R_2^{np+2} - R_1^{np+2}}{R_2^{-np+2} - R_1^{-np+2}}\right]$$

$$H_\varphi = -\sum_{n\ \text{odd}} \frac{2Jf}{\pi n(2-np)} \left(R_2^{-np+2} - R_1^{-np+2}\right) \rho^{np-1} \sin np\varphi$$

$$\left[1 + \frac{2-np}{2+np} \frac{1}{R_s^{2np}} \frac{R_2^{np+2} - R_1^{np+2}}{R_2^{-np+2} - R_1^{-np+2}}\right]$$

$R_1 < \rho < R_2$

$$H_\rho = \sum_{n\ \text{odd}} \frac{2Jf}{\pi n(2-np)} \left(R_2^{-np+2} - \rho^{-np+2}\right) \rho^{np-1} \cos np\varphi$$

$$\left[1 + \frac{2-np}{2+np} \frac{1}{R_s^{2np}} \frac{R_2^{np+2} - R_1^{np+2}}{R_2^{-np+2} - \rho^{-np+2}}\right]$$

$$+ \sum_{n\ \text{odd}} \frac{2Jf}{\pi n(2+np)} \frac{\left(\rho^{np+2} - R_1^{np+2}\right)}{\rho^{np+1}} \cos np\varphi \left[1 + \left(\frac{\rho}{R_s}\right)^{2np} \frac{R_2^{np+2} - R_1^{np+2}}{\rho^{np+2} - R_1^{np+2}}\right]$$

$$H_\varphi = -\sum_{n\ \text{odd}} \frac{2Jf}{\pi n(2-np)} \left(R_2^{-np+2} - \rho^{-np-1}\right) \rho^{np-1} \sin np\varphi$$

$$\left[1 + \frac{2-np}{2+np} \frac{1}{R_s^{2np}} \frac{R_2^{np+2} - R_1^{np+2}}{R_2^{-np+2} - \rho^{-np+2}}\right]$$

$$+ \sum_{n\ \text{odd}} \frac{2Jf}{\pi n(2+np)} \frac{\left(\rho^{np+2} - R_1^{np+2}\right)}{\rho^{np+1}} \sin np\varphi \left[1 - \left(\frac{\rho}{R_s}\right)^{2np} \frac{R_2^{np+2} - R_1^{np+2}}{\rho^{np+2} - R_1^{np+2}}\right]$$

$R_2 < \rho < R_s$

$$H_\rho = \sum_{n\ \mathrm{odd}} \frac{2J_f}{\pi n(np+2)} \frac{(R_2^{np+2} - R_1^{np+2})}{\rho^{np+1}} \cos np\varphi \left[1 + (\frac{\rho}{R_s})^{2np} \right]$$

$$H_\varphi = \sum_{n\ \mathrm{odd}} \frac{2J_f}{\pi n(np+2)} \frac{(R_2^{np+2} - R_1^{np+2})}{\rho^{np+1}} \sin np\varphi \left[1 - (\frac{\rho}{R_s})^{2np} \right]$$

$np = 2$

$\underline{\rho < R_1}$

$$H\rho = \frac{2Jf}{\pi} \ln \frac{R_2}{R_1} \rho \cos 2\varphi \left[1 + \frac{R_2^4 - R_1^4}{4R_s^4 \ln \frac{R_2}{R_1}} \right]$$

$$H\varphi = - \frac{2Jf}{\pi} \ln \frac{R_2}{R_1} \rho \sin 2\varphi \left[1 + \frac{R_2^4 - R_1^4}{4R_s^4 \ln \frac{R_2}{R_1}} \right]$$

$\underline{R_1 < \rho < R_2}$

$$H\rho = \frac{2Jf}{\pi} \ln \frac{R_2}{\rho} \rho \cos 2\varphi \left[1 + \frac{R_2^4 - R_1^4}{4R_s^4 \ln \frac{R_2}{R_1}} \right]$$

$$+ \frac{Jf}{2\pi} \frac{\rho^4 - R_1^4}{\rho^3} \cos 2\varphi \left[1 + \frac{R_2^4 - R_1^4}{\rho^4 - R_1^4} \left(\frac{\rho}{R_s}\right)^4 \right]$$

$$H\varphi = - \frac{2Jf}{\pi} \ln \frac{R_2}{\rho} \rho \sin 2\varphi \left[1 + \frac{R_2^4 - R_1^4}{4R_s^4 \ln \frac{R_2}{\rho}} \right]$$

$$+ \frac{Jf}{2\pi} \frac{\rho^4 - R_1^4}{\rho^3} \sin 2\varphi \left[1 - \frac{R_2^4 - R_1^4}{\rho^4 - R_1^4} \left(\frac{\rho}{R_s}\right)^4 \right]$$

$\underline{R_2 < \rho < R_s}$

$$H\rho = \frac{Jf}{2\pi} \frac{R_2^4 - R_1^4}{\rho^3} \cos 2\varphi \left[1 + \left(\frac{\rho}{R_s}\right)^4 \right]$$

$$H\varphi = \frac{Jf}{2\pi} \frac{R_2^4 - R_1^4}{\rho^3} \sin 2\varphi \left[1 - \left(\frac{\rho}{R_s}\right)^4 \right]$$

TABLE II

Field Intensity of Phase a of the Armature Winding with Shielding

$np \neq 2$

$\rho < R_i$

$$H_\rho = \sum_{n\ odd} \frac{2J_a \cos\frac{n\pi}{3}}{\pi n(2-np)} \frac{(R_o^{-np+2} - R_i^{-np+2})}{\rho^{-np+1}} \cos np\varphi \left(1 + \frac{(2-np)(R_o^{np+2} - R_i^{np+2})}{(2+np)R_s^{2np}(R_o^{-np+2} - R_i^{-np+2})}\right)$$

$$H_\varphi = -\sum_{n\ odd} \frac{2J_a \cos\frac{n\pi}{3}}{\pi n(2-np)} \frac{(R_o^{-np+2} - R_i^{-np+2})}{\rho^{-np+1}} \sin np\varphi \left(1 + \frac{(2-np)(R_o^{np+2} - R_i^{np+2})}{(2+np)R_s^{2np}(R_o^{-np+2} - R_i^{-np+2})}\right)$$

$R_i < \rho < R_o$

$$H_\rho = \sum_{n\ odd} \frac{2J_a \cos\frac{n\pi}{3}}{\pi n(2-np)} \frac{(R_o^{-np+2} - \rho^{-np+2})}{\rho^{-np+1}} \cos np\varphi \left(1 + \frac{(2-np)(R_o^{np+2} - R_i^{np+2})}{(2+np)R_s^{2np}(R_o^{-np+2} - \rho^{-np+2})}\right)$$

$$+ \sum_{n\ odd} \frac{2J_a \cos\frac{n\pi}{3}}{\pi n(2+np)} \frac{(\rho^{np+2} - R_i^{np+2})}{\rho^{np+1}} \cos np\varphi \left(1 + (\rho/R_s)^{2np} \frac{R_o^{np+2} - R_i^{np+2}}{\rho^{np+2} - R_i^{np+2}}\right)$$

$$H_\varphi = -\sum_{n\ odd} \frac{2J_a \cos\frac{n\pi}{3}}{\pi n(2-np)} \frac{(R_o^{-np+2} - \rho^{-np+2})}{\rho^{-np+1}} \sin np\varphi \left(1 + \frac{(2-np)(R_o^{np+2} - R_i^{np+2})}{(2+np)R_s^{2np}(R_o^{-np+2} - \rho^{-np+2})}\right)$$

$$+ \sum_{n\ odd} \frac{2J_a \cos\frac{n\pi}{3}}{\pi n(2+np)} \frac{(\rho^{np+2} - R_i^{np+2})}{\rho^{np+1}} \sin np\varphi \left(1 - (\rho/R_s)^{2np} \frac{R_o^{np+2} - R_i^{np+2}}{\rho^{np+2} - R_i^{np+2}}\right)$$

$R_o < \rho < R_s$

$$H_\rho = \sum_{n\ odd} \frac{2J_a \cos\frac{n\pi}{3}}{\pi n(2+np)} \frac{(R_o^{np+2} - R_i^{np+2})}{\rho^{np+1}} \cos np\varphi \left(1 + (\rho/R_s)^{2np}\right)$$

$$H_\varphi = \sum_{n\ odd} \frac{2J_a \cos\frac{n\pi}{3}}{\pi n(2+np)} \frac{(R_o^{np+2} - R_i^{np+2})}{\rho^{np+1}} \sin np\varphi \left(1 - (\rho/R_s)^{2np}\right)$$

$np = 2$

$\underline{\rho < R_i}$

$$H_\rho = \frac{Ja}{\pi} \ln(R_o/R_i)\ \rho\cos 2\varphi \left(1 + \frac{R_o^4 - R_i^4}{4R_s^4 \ln(R_o/R_i)}\right)$$

$$H_\varphi = -\frac{Ja}{\pi} \ln(R_o/R_i)\ \rho\sin 2\varphi \left(1 + \frac{R_o^4 - R_i^4}{4R_s^4 \ln(R_o/R_i)}\right)$$

$\underline{R_i < \rho < R_o}$

$$H_\rho = \frac{Ja}{\pi} \ln(R_o/\rho)\ \rho\cos 2\varphi \left(1 + \frac{R_o^4 - R_i^4}{4R_s^4 \ln(R_o/\rho)}\right) + \frac{Ja}{4\pi} \frac{\rho^4 - R_i^4}{\rho^3} \cos 2\varphi \left(1 + \frac{R_o^4 - R_i^4}{\rho^4 - R_i^4} (\rho/R_s)^4\right)$$

$$H_\varphi = -\frac{Ja}{\pi} \ln(R_o/\rho)\ \rho\sin 2\varphi \left(1 + \frac{R_o^4 - R_i^4}{4R_s \ln(R_o/\rho)}\right) + \frac{Ja}{4\pi} \frac{\rho^4 - R_i^4}{\rho^3} \sin 2\varphi \left(1 + \frac{R_o^4 - R_i^4}{\rho^4 - R_i^4} (\rho/R_s)^4\right)$$

$\underline{R_o < \rho < R_s}$

$$H_\rho = \frac{Ja}{4\pi} \frac{R_o^4 - R_i^4}{\rho^3} \cos 2\varphi \left(1 + (\rho/R_s)^4\right)$$

$$H_\varphi = \frac{Ja}{4\pi} \frac{R_o^4 - R_i^4}{\rho^3} \sin 2\varphi \left(1 - (\rho/R_s)^4\right)$$

TABLE III

Energy and Inductance Equations

Field Energy, $np \neq 2$

$$W_f = \frac{2\mu_0 J_f^2 R_2^4 \ell}{\pi n^3 p(4-n^2p^2)^2}\Big[8 - 4np - 2n^2p^2 + n^3p^3 + (4-3n^2p^2-n^3p^3)\, y^4$$

$$- (8-6n^2p^2)y^{np+2} - (4-4np+n^2p^2)y^{2np+4}$$

$$+ \frac{2(2-np)}{z^{2np}}\left(2+2np - (6+np)y^{np+2} - (2+np)y^{2np} + (4-np)y^{2np+4} + (2+np)y^{3np+2} \right)$$

$$+ \frac{2(2-np)^2}{z^{4np}}\left(3 - 6y^{np+2} - 3y^{2np} + 3y^{2np+4} + 6y^{3np+2} - 3y^{4np+4} \right)\Big]$$

$np = 2$

$$W_f = \mu_0 \frac{J_f^2 R_2^4 \ell}{\pi}\left(y^4 \ln y + \frac{1}{4} - \frac{3y^4}{8} + \frac{y^8}{8} + \frac{16-17y^4+y^{12}}{16z^4} + \frac{6-19y^4+20y^8-7y^{12}}{16z^8} \right)$$

Armature Field Energy, $np \neq 2$

$$W_a = \frac{2\mu_0 J_a^2 R_0^2 \ell \cos^2 \frac{n\pi}{3}}{\pi n^3 p(4-n^2p^2)^2}\Big[8-4np-2n^2p^2+n^3p^3+(4-3n^2p^2-n^3p^3)x^4-(8-6n^2p^2)x^{np+2}$$

$$-(4-4np+n^2p^2)x^{2np+4} + \frac{2(2-np)}{w^{2np}}\left(2+2np-(6+np)x^{np+2}-(2+np)x^{2np}+(4-np)x^{2np+4} +(2+np)x^{3np+2} \right)$$

$$+ \frac{2(2-np)^2}{w^{4np}}\left(3-6x^{np+2}-3x^{2np}+3x^{2np+4}+6x^{3np+2}-3x^{4np+4} \right)\Big]$$

$np = 2$

$$W_a = \mu_0 \frac{J_a^2 R_0^2 \ell}{4\pi}\left(x^4 \ln x + \frac{1}{4} - \frac{3x^4}{8} + \frac{x^8}{8} + \frac{16-17x^4+x^{12}}{16w^4} + \frac{6-19x^4+20x^8-7x^{12}}{16w^8} \right)$$

Field Self Inductance, $np \neq 2$

$$L_f = \frac{16\mu_o N_f^2 \ell}{\pi^3 n^3 p(4-n^2p^2)^2(1-y^2)^2} \left[\text{Same as for field energy} \right]$$

$np = 2$

$$L_f = \frac{4\mu_o N_f^2 \ell}{\pi^3 (1-y^2)^2} \left[\text{Same as for field energy} \right]$$

Armature Self Inductance, $np \neq 2$

$$L_a = \frac{144\mu_o N_a^2 \ell \cos^2 \frac{n\pi}{3}}{\pi^3 n^3 p(4-n^2p^2)^2(1-x^2)^2} \left[\text{Same as for armature field energy} \right]$$

$np = 2$

$$L_a = \frac{9\mu_o N_a^2 \ell}{\pi^3(1-x^2)^2} \left[\text{Same as for armature field energy} \right]$$

Mutual Inductance, $np \neq 2$

$$L_{af} = \frac{96\mu_o N_a N_f \ell}{\pi^3(1-x^2)(1-y^2)} \sum_{n \text{ odd}} \frac{(R_2/R_o)^{np}(1-y^{np+2})}{n^3 p(np+2)} \left[\frac{1-x^{-p+2}}{2-np} + \frac{1-x^{np+2}}{(2+np)w^{2np}}\right] \cos\frac{n\pi}{3} \cos np\varphi$$

$np = 2$

$$L_{af} = \frac{6\mu_o N_a N_f \ell(1+y^2)}{\pi^3(1-x^2)}(R_2/R_o)^2\left[\ln\frac{1}{x} + \frac{1-x^4}{4w^4}\right] \cos 2\varphi$$

Magnitude of First Order Mutual Inductance, $np \neq 2$

$$M = \frac{48\mu_o N_a N_f \ell}{\pi^3 p(p+2)} (R_2/R_o)^p \frac{1-y^{p+2}}{(1-x^2)(1-y^2)} \left[\frac{1-x^{-p+2}}{2-p} + \frac{1-x^{p+2}}{(2+p)w^{2p}} \right]$$

$np = 2$

$$M = \frac{6\mu_o N_a N_f \ell(1-y^2)}{\pi^3(1-x^2)}(R_2/R_o)^2 \left[\ln\frac{1}{x} + \frac{1-x^4}{4w^4} \right]$$

III. MACHINE DESIGN

The procedure used for machine design is essentially the same as that used in reference 1. It is assumed that the iron in the shield does not appreciably affect the open circuit armature voltage other than through the effect of M. The following additional assumptions are made. The inner radius of the shield is equal to the outer radius of the armature, i.e. $w=1$. The inner radius of the armature is 3 cm greater than the outer radius of the field to allow for the insulating dewar and clearance for rotation. The current densities are $J_f = 10^8$ amps/m^2 and $J_a = 10^6$ amps/m^2. The magnetic permeability of the iron shield is infinite and the permeability of the rest of the machine is about the same as air. The basic equations used for estimating the machine size and output power are

$$P = 3/2\, V_a I_a \tag{III-1}$$

$$V_a = \omega M I_f \tag{III-2}$$

$$N_a I_a = \frac{\pi(R_o^2 - R_i^2)J_a}{6} \tag{III-3}$$

The weight of the shield is calculated on the density of iron, 7.87 grams/cm^3. The field and armature weights are figured on the basis of 6 grams/cm^3 with an allowance of .25 times their weight for a shaft and support. [1] The length, l, of the machine is taken as twice the diameter of the armature. This is used as a rough figure based on existing machines. The ratio to obtain the maximum power output for a given volume is a problem of optimization which will not be considered in this paper. This length is the length of the straight portion of the windings. To this must be added the length due to the end turns to get the overall length of the machine

and shield. The end turn length is calculated on the size of the semi-circle that the windings must make to go from one pole to the next.

The shaft is hollow and is designed on the basis of the formula

$$\tau = \frac{321{,}000\ H\ d}{n\ (d^4 - d_1^4)} \qquad \text{(III-4)}$$

with H in horsepower, d in inches, n in rpm and τ in psi. Mild steel will fail with a value of τ equal to about 65,000 psi in torsion only. [6]

Heat transfer to the supercooled region is through the insulating dewar* and through the shaft which will carry all other necessary leads into the supercooled region. The heat loss is calculated from

$$w = \bar{k}\ A\ \frac{dT}{dt} \qquad \text{(III-5)}$$

with $\bar{k}$ equal to the average heat transfer coefficient, A equal to the area through which the heat is being transferred, and dT/dt equals the average thermal gradient.

* An evacuated insulation, named after the inventor.

RESULTS

The machine model and parameters for the shielded, superconducting field, alternator were developed in Chapter II and Appendix A. A 27,200 kw (36,500 HP) generator was designed in Appendix B. Table IV shows a summary of the results.

TABLE IV

A 3 phase, 3 pole-pair, generator of 27,200 kw.

Prime mover speed			9000 rpm
Electrical speed			450 cps
Power output			27,200 kw 36,500 hp
Armature power lost			3.5 kw
Reactive armature power			65 kw
Field power lost			negligible
Heat loss at 4.2°K			14 watts
at 70°K			393 watts
Shaft	outside diameter		12.7 cm
	inside diameter		10.2 cm
	(Shaft will carry the power leads and coolants.)		
Alternator diameter			.754 m
	length	electrical	1.2 m
		physical	1.67 m
Shield	outside diameter		.754 m
	inside diameter		.60 m

Armature	outside diameter	.60 m
	inside diameter	.48 m
Field	outside diameter	.42 m
	inside diameter	.22 m
Weights	Shield	2.13 tons
	Armature	1.02 tons
	Field	1.00 tons
	Shaft, end support, etc.	.50 tons
	Total	4.65 tons
Specific horsepower		7840 hp/ton

DISCUSSION OF RESULTS

The figures for the 27,200 kw generator are the best achieved in an attempt to develop an optimization procedure. As a result, only a generator was designed. The complexity of the problem did not allow adequate determination of an optimization procedure in the time allowed.

The output voltage was assumed to be equal to the open circuit voltage. The armature reaction is sufficiently small so that the error introduced is negligible. At a higher frequency and a different number of pole-pairs, it may have more effect.

A small amount of power is lost in the armature due to the resistance of the copper wire. No significant power is lost in the field. This is one advantage of using a superconducting field.

The dewar is composed of 1 cm of vacuum insulation around the superconductor windings, a 1 cm space for liquid nitrogen to flow through and a 1 cm vacuum insulation to separate the liquid nitrogen from the atmosphere. The use of liquid nitrogen as an intermediate stage of cooling lowers the heat loss at $4.2^{o}K$. It is cheaper, in terms of coolant plant power, to remove as much of the heat as possible at as high a temperature as is practical. The coolants will flow into the dewar through the shaft. Centrifugal force and the difference in density between the liquid and gas aids the flow of the coolants. The heat loss through the dewar is calculated on the basis of 0.01 mm Hg pressure in the dewar. The heat loss through the shaft is hard to calculate as the heat transfer problem is not clearly defined. The heat loss at $4.2^{o}K$ through the shaft was calculated using an effective length of 35 cm between the $4.2^{o}K$ portion

of the shaft and the 70°K portion of the shaft. An effective length of 10 cm was used for calculating the shaft heat loss at 70°K.

The shaft carries the coolants and the power leads for the field. A small amount of power can be supplied to the field from time to time to boost it back to the desired level. This requirement is small and should cause no problem with using the shaft to bring to leads into the dewar.

The speed of 9000 rpm was used to allow the prime mover to be either a steam turbine or a gas turbine. Both can operate at this speed efficiently.

CONCLUSIONS AND RECOMMENDATIONS

Shipboard electrical propulsion systems can be greatly reduced in size by the use of superconductors. A conventional electric propulsion plant for 35,000 shaft horsepower weighs 275 tons and a reduction gear-shaft drive weighs 68 tons. [2]

An opti mization formula was not successfully derived, but the generator described indicates the reduction in size and weight possible. It will easily be competitive with standard shafting drives. The figures described have been achieved by manufacturers now. These do not make any demands on future technology.

It is recommended that the rest of the system be analysed and that another attempt be made to find a procedure to optimize the design of an alternator.

Several individual items need attention. A fully superconducting machine needs to be considered. Additional size and weight may be gained. The problem of AC losses has to be taken care of if it is to be considered.

A structural analysis of turbine and generator need to be done to determine the maximum speed available, if the speed can be increased, the size and weight can be decreased.

The power must be transferred from the generator to the control system and to the motor. A method must be used which will keep the size, weight, and power loss at a minimum.

A control system must be designed. A cycloconverter seems to have application in this system as it offers the wide variation in control desired.

The entire system will have to be designed from a thermodynamics point of view to minimize the coolant plant which will have to be provided. It should be possible to reduce the heat losses in the alternator by a better thermodynamic design.

Upon completion of the design of the system a working model will be needed to prove the feasibility for shipboard use.

APPENDIX

A. DEVELOPMENT OF FIELD AND ENERGY EQUATIONS AND INDUCTANCE PARAMETERS

Surface current

A cylinder of surface current is formed by a differential element dr thick at radius r. This is shown in Figure 2 and Figure 3. This can be represented by

$$K(r) = \sum_{n \text{ odd}} b_n \sin \frac{n\pi x}{L}$$

where

$$b_n = \frac{1}{L} \int_{-L}^{L} f(X) \sin \frac{n\pi x}{L}$$

with

$$x = \varphi$$

$$L = \frac{\pi}{p}$$

$$\begin{aligned} f(x) &= J_f dr \qquad & 0 < x < L \\ &= -J_f dr \qquad & -L < x < 0 \end{aligned}$$

evaluating

$$b_n = \frac{4 J_f dr}{n\pi}$$

and

$$K(r) = \sum_{n \text{ odd}} \frac{4 J_f dr}{n\pi} \sin np\varphi$$

Fields

The fields are the solution to Laplace's Equation

$$\nabla^2 U = 0$$

subject to the boundary conditions as stated in Chapter II. The solution

is

$$U = R(r)\Phi(\varphi)$$

$$U = (A\rho^{q} + B\rho^{-q})(C \sin q\varphi + D \cos q\varphi)$$

The field inside of the alternator is composed of a superposition of six solutions. There is one solution for the inside of the ring of field current one for the outside, and one for the shield. There is a corresponding set for the armature. The fields are of two basic forms, one for the radius to the point in question being inside the ring of current ($\rho < r$) and the other for the radius to the point in question being outside the ring of current ($\rho > r$). The current rings are then integrated from R_1 to R_2 and from R_i to R_o. This removes the r dependence of the solution. For a point in the winding, one solution has to be integrated out to ρ from the inside of the winding and the other has to be integrated from ρ to the outside of the winding.

<u>Energy</u>

The energy in the fields has to be integrated in pieces corresponding to the different radii. The integrations are straight forward if the similarity of the pieces of the integrals is used. Upon completion of the integration the radius ratios are substituted and the results are as shown in Table II. There is no difference in the procedure for the armature.

<u>Inductance</u>

Self inductance is obtained from the relation

$$L_f = \frac{2W_f}{I^2}$$

with

$$I_f = \frac{\pi(R_2^2 - R_1^2)\, J_f}{2N_f}$$

Mutual inductance is calculated from the turns linked in an elemental coil of the armature by the field. An elemental coil is

$$\frac{6\ N_a}{\pi(R_o^2 - R_i^2)}\ \rho d\rho d\varphi$$

so that

$$d^2\lambda = 6\ N_a\ \mu_o\ \rho d\rho d\varphi \int_{\varphi - \frac{\pi}{p}}^{\varphi} H_\rho(\rho,\psi)\ \rho d\psi$$

$$H_\rho(\rho,\psi) = \sum_{n\ \text{odd}} \frac{2J_f}{\pi n(2+np)}\ \frac{R_2^{np+2} - R_1^{np+2}}{\rho^{np+1}}\ \cos np\psi\ \left(1+ \left(\frac{\rho}{R_s}\right)^{2np}\right)$$

Substitute for H_ρ and J_f and carry out an integration over ψ and then over ρ and φ. Then $L_{af} = \frac{\lambda}{I_f}$, which is in Table III. M is the magnitude of the first order component of L_{af}. It is the primary determining factor in the voltage generated by the alternator.

B. MACHINE DESIGN

The voltage produced by the alternator is

$$V_a = \omega M I_f$$

The current is

$$I_a = \frac{\pi(R_o^2 - R_i^2)J_a}{6N_a}$$

Therefore the power is

$$P = 3/2\ V_a I_a$$

$$= \frac{\omega M I_f \pi (R_o^2 - R_i^2) J_a}{4Na}$$

Substitute M from Table III.

$$P = \frac{12\mu_o \omega I_f N_f R_o^2 J_a \ell}{p(p+2)} \left(\frac{R_2}{R_o}\right)^p \frac{1 - y^{p+2}}{1 - y^2} \left[\frac{1 - x^{-p+2}}{2 - p} + \frac{1 - x^{p+2}}{(2 + p)W^{2p}} \right]$$

$$I_f N_f = \frac{\pi R_2^2 (1 - y^2) J_f}{2}$$

$$W = 1$$

$$\omega = p\ \omega_m$$

$$\ell = 4R_o$$

Substituting the above into the formula for P

$$P = \frac{96\ \omega_m\ 10^7\ R_o^5}{p + 2} \left[\frac{R_2}{R_o}\right]^{p+2} (1 - y^{p+2}) \left[\frac{1 - x^{-p+2}}{2 - p} + \frac{1 - x^{p+2}}{2 + p} \right] \tag{B-1}$$

Since the shield is made of iron the magnetic field intensity is limited to 2 Webers/m^2 at the shield. The contribution of the armature to the intensity is $\frac{1}{200}\left(\frac{R_o}{R_2}\right)^{p+2}$ in the first harmonic. The contribution would be about 10% in an 11 pole-pair machine. A first estimate of field intensity can be made by neglecting the armature contribution. All higher order harmonics are negligible, decreasing on the order of $\frac{1}{p+2}\left(\frac{R_2}{R_o}\right)^{p+2}$.

The intensity due to the field is

$$B_\rho = \frac{2Jf\mu_o}{\pi(2+p)} \frac{R_2^{p+2} - R_1^{p+2}}{\rho^{p+1}} \cos p\varphi \left[1 + \left(\frac{\rho}{R_s}\right)^{2p}\right]$$

$$\rho = R_s = R_o$$

The maximum intensity is desired so $\varphi = 0$. Evaluating the constants

$$|B_\rho| = \frac{160\ R_o}{2+p} \left(\frac{R_2}{R_o}\right)^{p+2} (1 - y^{p+2}) \tag{B-2}$$

The following constraints due to the physical arrangement of the problem

$$R_1 > r_{shaft}$$

$$0 < y < 1$$

$$R_i = R_2 + .03m$$

$$0 < x < 1$$

$$R_2 = R_1/y$$

$$R_o = R_i/x$$

used with equations B-1 and B-2 are sufficient to design an alternator.

The size of the shield is determined by the maximum flux that the shield is to carry.

$$B = |B_\rho| \cos p\varphi \qquad \text{at} \quad \rho = R_o$$

The flux per meter length

$$= \int_0^{\frac{\pi}{2p}} |B_\rho| \cos p\varphi\, R_o\, d\varphi$$

$$= \frac{|B_\rho|\ R_o}{p}$$

The maximum flux density allowed in the shield is 2 Webers/m^2 so that the iron does not saturate. The shield then must have a thickness

$$t = \frac{|B_\rho|\ R_o}{2p}$$

and the machine outside radius is

$$R_{so} = \left(1 + \frac{|B_\rho|}{2p}\right) R_o$$

Several attempts were made to derive an optimization formula from the above information with no success. The best machine designed in the process is the one for which the results are given. The magnitude of the magnetic field intensity at the shield was 1.54 Webers/m^2.

The shaft size was calculated from the formula

$$\tau = \frac{321,000\ H\ d}{n(d^4 - d_1^4)}$$

$$H = 36,500 \ \text{hp}$$

$$n = 9000 \ \text{rpm}$$

$$d = 5 \ \text{in.}$$

$$d_1 = 4 \ \text{in.}$$

with the resultant torsional stress

$$\tau = 17,500 \ \text{psi}$$

This value leaves a large margin of safety as all steels tested failed above 50,000 psi. [6] The only restraints on the shaft size are that it be smaller than the inside radius of the field and large enough to hold the leads and carry the coolants.

The heat loss is calculated with the formula

$$W = \bar{k}\ A\ \frac{dT}{dt}$$

and an idea of the geometry of the problem. Heat transfer through the dewar is handled in two steps. The first step is from 4.2°K to 70°K. That is the temperature of liquid nitrogen, which will be used as an intermediate coolant to reduce the heat loss at 4.2°K. The second step is from 70°K to 300°K.

The dewar is approximated by a cylinder slightly larger than the outer radius of the field and two disks at the ends of the field.

$$A = 2\pi R_{dewar} \left(\ell + \frac{\pi R_2}{p}\right) + 2\pi \left(R_{dewar}^2 - R_{shaft}^2\right)$$

The following values were used for the inner dewar

$$R_{dewar} = 22 \text{ cm}$$

$$\ell = 120 \text{ cm}$$

$$R_2 = 21 \text{ cm}$$

$$p = 3$$

$$R_{shaft} = 6.35 \text{ cm}$$

The area calculated was 22,500 cm^2. A similar calculation for the outer dewar yielded an area equal to 25,070 cm^2.

The values of the heat transfer coefficients used were taken as that of silica aerogel in a dewar with a pressure of 10^{-2} mm Hg. Better values are obtainable, but are calculated for a dewar at least 1 inche thick. Dewars thinner than 1 inch have a greater heat transfer coefficient than that which is listed for that type of dewar. The values used are 5 microwatts/cm °K for the dewar between 4.2°K and 70°K, and 30 microwatts/cm °K for the outer dewar. Each dewar is 1 cm thick with a 1 cm space between for the liquid nitrogen. The heat losses calculated are 7.4 watts for the inner dewar and 173 watts for the outer dewar. These figures assume that the ends are the same thickness as the cylinders.

The heat loss due to the shaft is harder to estimate. The superconducting portion of the machine is about 145 cm long. The supports for the field are assumed to be evenly distributed along the shaft and the shaft and supports are assumed to be insulated so that the heat that flows into the dewar through the shaft must come in the supports. Then the average path length of heat conduction is at least 35 cm from 4.2°K to the point where the liquid nitrogen passes through the shaft, which is at 70°K. The conductivity assumed is that of type 347 stainless steel, about 40 milliwatts/cm °K between 4.2°K and 70°K, and about 110 milliwatts/cm °K between 70°K and 300°K. The cross sectional area of

the shaft is 43 cm^2. The heat loss calculated is 3.3 watts from each end of the shaft. The heat flow from 70°K to 300°K is estimated on a length of 10 cm, assuming that the shaft will be insulated for a short distance beyond the dewar. The heat loss at 70°K is 110 watts from each end of the shaft. The total heat loss at 4.2°K is 14 watts and 393 watts at 70°K.

The power loss in the armature due to resistance is easily calculated.

$$P = I^2 R$$

$$= J_a^2 \rho \ell A$$

$$J_a = 100 \text{ amps/cm}^2$$

$$\rho = 1.9 \text{ micro-ohm-cm}$$

$$\ell = 167 \text{ cm}$$

$$A = 1100 \text{ cm}^2$$

The resultant power loss is 3.5 kw.

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